F. SCHIPP, W. R. WADE, AND P. SIMON WITH ASSISTANCE FROM J. PÁL, *Walsh Series, An Introduction to Dyadic Harmonic Analysis*, Akadémiai Kiadó, 1990, 560 pp.

Simple examples of the Walsh functions are the 1-periodic function r and its dyadic dilations $r(2^n \cdot)$, where $r(x) = 1$ for $x \in [0, \frac{1}{2})$ and $r(x) = -1$ for $x \in [\frac{1}{2}, 1)$. These functions are very useful in many applications, typically in data signal processing. This book gives a thorough introduction to the theory of Walsh–Fourier analysis. It starts with two chapters of basics, including the Walsh functions, the dyadic group, the dyadic derivative, the Walsh–Fourier coefficients and series, and convergence and summability. Chapter 3 is about dyadic martingales, Hardy spaces, and various dyadic maximal functions. It ends with the proof of the almost everywhere convergence of Walsh–Fourier series of L^p $(p > 1)$ functions. Chapter 6 provides other sufficient conditions for almost everywhere convergence and summability. Norm convergence and the set of divergence of Walsh–Fourier series are examined in Chapter 4, where one can also find a proof of the existence of an L^1 function with almost everywhere divergent Walsh–Fourier series. Walsh polynomial approximation and the study of bases make up Chapter 5. Results about uniqueness and representation may be found in Chapters 7 and 8. The book ends with a chapter on the Walsh–Fourier transform. Each chapter contains exercises and historical comments.

JOHN ZHANG

J. SZABADOS AND P. VÉRTESI, *Interpolation of Functions*, World Scientific, 1990, 305 pp.

As can be expected, given the authors and the title of the book, this is a very careful and exhaustive treatment of interpolation by univariate polynomials, mostly algebraic, with the usual connections to the trigonometric case. It is the proverbial book nobody in the area can do without. The chapter headings tell the story:

I. Lagrange Interpolation;
II. Some Convergent Interpolatory Processes;
III. The Lebesgue Function and Lebesgue Functions-Type Sums;
IV. Divergence of Lagrange Interpolation;
V. Hermite-Féjer and Hermite-Féjer Type Interpolations;
VI. Comparison of Lagrange and Hermite-Féjer Interpolations;
VII. Some Problems in the Theory of Lacunary Interpolation;
VIII. Miscellaneous Problems;
IX. Appendix: Some Frequently Used Relations and Theorems.

Each chapter ends with a section on "Problems and results" or "Problems and remarks." The discussion is almost entirely in the uniform metric. Survey books such as this could be even more useful if each item in the bibliography were to include the page number(s) at which the item is cited.

CARL DE BOOR

B. GOLUBOV, A. EFIMOV, AND V. SKVORTSOV, *Walsh Series and Transforms*, Kluwer Academic, 1991, 368 pp.

Orthogonal systems (such as orthogonal polynomials, trigonometric series, and wavelets), their theories, and applications have been intensively studied. The Walsh functions, discovered in 1923, are suitable for signal processing, image processing, and many related areas because of their rectangular shape. This book gives a systematic treatment of the Walsh–Fourier series and transforms, and some of their applications. Chapters 1–5 and Chaps. 7–9 deal with the theory of Walsh–Fourier series (basic properties, uniqueness, $(C, 1)$ sums, uses of the Hardy–Littlewood maximal operator, convergence in L^p, etc.). Chapter 10 covers approximations by Walsh systems. Multiplicative transforms and their applications may be found in Chaps. 6, 11, and 12. The appendices are quite handy as they provide much useful background information. This makes the book accessible to a wide audience, which could certainly include beginning graduate students.

JOHN ZHANG

A. ISERLES AND S. P. NØRSETT, *Order Stars*, Chapman & Hall, 1991, 248 pp.

Numerical algorithms for solving differential equations (ordinary and partial) require at least two important properties: they have to converge to the actual solution with a certain order and they have to be stable in the sense that a local change should not affect the solution globally. Quite often the numerical method is completely determined by a rational function R. The order of the numerical method can then be obtained by the degree of interpolation of R to a certain function f and stability is equivalent to a condition on the ratio R/f in a portion of the complex plane. Order stars give a technique for examining order and stability. Let f be a meromorphic function (a finite number of essential singularities are also allowed) and let R be a rational approximation to f. If $\mathbf{C}^* = \mathbf{C} \cup \{\infty\}$, then the order star of $\{f, R\}$ is the triplet $\{\mathscr{A}_+, \mathscr{A}_0, \mathscr{A}_-\}$ where $\mathscr{A}_+ = \{z \in \mathbf{C}^*: |R(z)/f(z)| > 1\}$, $\mathscr{A}_0 = \{z \in \mathbf{C}^*: |R(z)/f(z)| = 1\}$, and $\mathscr{A}_- = \{z \in \mathbf{C}^*: |R(z)/f(z)| < 1\}$. Information about interpolation points is in $\mathscr{A}_0$; information about zeros of R and poles of f is in $\mathscr{A}_-$; poles of R and zeros of f are to be found in $\mathscr{A}_+$. Particular attention is paid to rational approximants for the exponential function because Runge–Kutta methods and Obrechkoff methods for solving $y' = f(t, y)$ for $t \in [a, b]$ and $y(a) = y_0$ naturally lead to such approximants. Padé approximants are very natural here because they maximize the order of approximation at one point. However, quite often maximal order does not imply stability, so there has to be an interaction between trying to achieve high order while maintaining stability. One way to achieve this is to put restrictions on the zeros or poles of the approximants, which leads to z-restricted Padé approximants (restrictions on the zeros) and p-restricted Padé approximants (restrictions on the poles). Order stars are also used to analyze partial differential equations, especially the advection equation $\partial u/\partial t = \partial u/\partial x$ and the diffusion equation $\partial u/\partial t = c\, \partial^2 u/\partial x^2$ $(c > 0)$. Order stars can also give interesting results in the theory of rational approximation. It is well known that the Padé tableaux for Padé approximants to a function f consists of square blocks, such that every block contains the same approximant for f. An important number is the maximal block size $\beta(f)$ in the Padé tableau for f. The authors use order stars to find some useful upper bounds on $\beta(f)$. Order stars are also used to give some results on approximants that map an open set into the open unit disc (contractive approximation). As an example the authors give a brief outline of the Pick–Nevanlinna interpolation problem. The book is nicely illustrated with many pictures of order stars, which in the spirit of the topic is very helpful and an absolute necessity. Indeed, it is by looking at order stars that many properties of the corresponding numerical method or approximation problem are revealed. Also very helpful is a description of 13 open problems. A nice piece of mathematics on the interaction between theory and practice.

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